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Spherical Images of Special Smarandache Curves in E^3

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Abstract: In this study, we introduce the spherical images of some special Smarandache curves according to Frenet frame and Darboux frame in E^3. Besides, we give some differential geometric properties of Smarandache curves and their spherical images.

Key Words: Smarandache curves, S.Frenet frame, Darboux frame, Spherical image.

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§1. Introduction

Curves especially regular curves are used in many fields such as CAGD, mechanics, kinematics and differential geometry. Researchers are used various curves in these fields. Special Smarandache curves are one of them. A regular curve in Minkowski spacetime, whose position vector is composed by Frenet frame vectors on another regular curve, is called a Smarandache curve ([7]). Some authors have studied on special Smarandache curves ([1, 2, 7]).

In this paper, we give the spherical images of some special Smarandache curves according to Frenet frame and Darboux frame in E^3. Also, we give some relations between the arc length parameters of Smarandache curves and their spherical images.

§2. Preliminaries

Let $\alpha(s)$ be an unit speed curve that satisfies $||\alpha^{'}(s)|| = 1$ in E^3. S.Frenet frame of this curve in E^3 parameterized by arc length parameter s is,

$$\alpha^{'}(s) = T, \qquad \frac{T^{'}(s)}{||T^{'}(s)||} = N(s), \qquad T(s) \times N(s) = B(s),$$

where $T(s)$ is the unit tangent vector, $N(s)$ is the unit principal normal vector and $B(s)$ is the

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unit binormal vector of the curve $\alpha(s)$. The derivative formulas of S.Frenet are,

$$\begin{bmatrix} T' \\ N' \\ B' \end{bmatrix} = \begin{bmatrix} 0 & \kappa & \tau \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{bmatrix} \begin{bmatrix} T \\ N \\ B \end{bmatrix}, \tag{1}$$

where $\kappa = \kappa(s) = ||T'(s)||$ and $\tau = \tau(s) = ||B'(s)||$ are the curvature and the torsion of the curve $\alpha(s)$ at s, respectively [4].

Let S be a regular surface and a curve $\alpha(s)$ be on the surface S. Since the curve $\alpha(s)$ is also a space curve, the curve $\alpha(s)$ has S.Frenet frame as mentioned above. On the other hand, since the curve $\alpha(s)$ lies on the surface S, there exists another frame which is called Darboux frame $\{T, g, n\}$ of the curve $\alpha(s)$. T is the unit tangent vector of the curve $\alpha(s)$, n is the unit normal of the surface S and g is a unit vector given by $g = n \times T$.The derivative formulas of Darboux frame are

$$\begin{bmatrix} T' \\ g' \\ n' \end{bmatrix} = \begin{bmatrix} 0 & \kappa_g & \kappa_n \\ -\kappa_g & 0 & \tau_g \\ -\kappa_n & -\tau_g & 0 \end{bmatrix} \begin{bmatrix} T \\ g \\ n \end{bmatrix}, \tag{2}$$

where, κ_g is the geodesic curvature, κ_n is the normal curvature and τ_g is the geodesic torsion of the curve $\alpha(s)$. The Darboux vector and the unit Darboux vector of this curve are given, respectively as follows

$$d = \tau_g T + \kappa_n g + \kappa_g n$$

$$c = \frac{d}{||d||} = \frac{\tau_g T + \kappa_n g + \kappa_g n}{\sqrt{\tau_g^2 + \kappa_n^2 + \kappa_g^2}}. \tag{3}$$

(1) $\alpha(s)$ is a geodesic curve if and only if κ_g=0.

(2) $\alpha(s)$ is an asymptotic line if and only if κ_n=0.

(3) $\alpha(s)$ is a principal line if and only if τ_g =0 ([6]).

The sphere in E^3 with the radius $r > 0$ and the center in the origin is defined by [3]

$$S^2 = \left\{ x = (x_1, x_2, x_3) \in E^3 : \langle x, x \rangle = r^2 \right\}.$$

Let the vectors of the moving frame of a curve $\alpha(s)$ with non-vanishing curvature are given. Assume that these vectors undergo a parallel displacement and become bound at the origin O of the Cartesian coordinate system in space. Then the terminal points of these vectors $T(s)$, $N(s)$ and $B(s)$ lie on the unit sphere S which are called the tangent indicatrix, the principal normal indicatrix and the binormal indicatrix, respectively of the curve $\alpha(s)$.

The linear elements ds_T, ds_N and ds_B of these indicatrices or spherical images can be easily obtained by means of (1). Since $T(s)$, $N(s)$ and $B(s)$ are the vector functions representing these curves we find

$$\begin{cases} ds_T^2 = \kappa^2 ds^2, \\ ds_N^2 = \left(\kappa^2 + \tau^2\right) ds^2, \\ ds_B^2 = \tau^2 ds^2. \end{cases} \tag{4}$$

Curvature and torsion appear here as quotients of linear elements; choosing the orientation of the spherical image by the orientation of the curve $\alpha(s)$ we have from (4)

$$\kappa = \frac{ds_T}{ds}, \qquad |\tau| = \frac{ds_B}{ds}. \tag{5}$$

Moreover, from (5) we obtain the Equation of Lancret ([5])

$$ds_N^2 = ds_T^2 + ds_B^2. \tag{6}$$

§3. Special Smarandache Curves According to S.Frenet Frame In E^3

3.1 TN- Smarandache Curves

Let $\alpha(s)$ be a unit speed regular curve in E^3 and $\{T, N, B\}$ be its moving S.Frenet frame. A Smarandache TN curve is defined by ([1])

$$\beta\left(s^*\right) = \frac{1}{\sqrt{2}}\left(T + N\right). \tag{7}$$

Let moving S. Frenet frame of this curve be $\{T^*, N^*, B^*\}$.

3.1.1 Spherical Image of the Unit Vector T_β^*

We can find the relation between the arc length parameters ds^* and ds as follows

$$\frac{ds^*}{ds} = \sqrt{\frac{2\kappa^2 + \tau^2}{2}}. \tag{8}$$

From the equations (5) and (8) we have

$$ds^* = \frac{\sqrt{2\kappa^2 + \tau^2}}{\kappa\sqrt{2}} ds_T. \tag{9}$$

From the equation (5) we obtain the spherical image of the unit vector T_β^* as

$$\frac{ds_T^*}{ds^*} = \kappa^* = \frac{\sqrt{2}\sqrt{\delta^2 + \mu^2 + \eta^2}}{\left(\sqrt{2\kappa^2 + \tau^2}\right)^2}, \tag{10}$$

where

$$\kappa^* = \frac{\sqrt{2}\sqrt{\delta^2 + \mu^2 + \eta^2}}{\left(\sqrt{2\kappa^2 + \tau^2}\right)^2}. \tag{11}$$

Here ([1]),

$$\begin{cases}\delta=-\left[\kappa^{2}\left(2\kappa^{2}+\tau^{2}\right)+\tau\left(\tau\kappa^{'}-\kappa\tau^{'}\right)\right],\\ \mu=-\left[\kappa^{2}\left(2\kappa^{2}+3\tau^{2}\right)+\tau\left(\tau^{3}-\tau\kappa^{'}+\kappa\tau^{'}\right)\right],\\ \eta=\kappa\left[\tau\left(2\kappa^{2}+\tau^{2}\right)-2\left(\tau\kappa^{'}-\kappa\tau^{'}\right)\right].\end{cases}$$

Then, from the equations (9) and (10)

$$ds_{T}^{*}=\frac{\sqrt{\delta^{2}+\mu^{2}+\eta^{2}}}{\kappa\left(2\kappa^{2}+\tau^{2}\right)^{3/2}}ds_{T} \tag{12}$$

is obtained.

3.1.2 Spherical Image of the Unit Vector N_{β}^{*}

If we use the equation (6) we have

$$\frac{ds_{N}^{*}}{ds^{*}}=\sqrt{\left(\kappa^{*}\right)^{2}+\left(\tau^{*}\right)^{2}}. \tag{13}$$

Besides, from the equations (6), (8) and (13)

$$ds_{N}^{*}=\sqrt{\left(\kappa^{*}\right)^{2}+\left(\tau^{*}\right)^{2}}\frac{\sqrt{2\kappa^{2}+\tau^{2}}}{\sqrt{2}\sqrt{\kappa^{2}+\tau^{2}}}ds_{N} \tag{14}$$

is obtained, where

$$\tau^{*}=\frac{\sqrt{2}\left[\left(\kappa^{2}+\tau^{2}-\kappa^{'}\right)\left(\kappa\sigma+\tau\omega\right)+\kappa\left(\kappa\tau+\tau^{'}\right)\left(\phi-\omega\right)+\left(\kappa^{2}+\kappa^{'}\right)\left(\kappa\sigma-\tau\phi\right)\right]}{\left[\tau\left(2\kappa^{2}+\tau^{2}\right)+\kappa^{'}\tau-\kappa\tau^{'}\right]^{2}+\left(\tau\kappa^{'}-\kappa\tau^{'}\right)^{2}+\left(2\kappa^{3}+\kappa\tau^{2}\right)^{2}} \tag{15}$$

and

$$\begin{cases}\omega=\kappa^{3}+\kappa\left(\tau^{2}-3\kappa^{'}\right)-\kappa^{''},\\ \phi=-\kappa^{3}-\kappa\left(\tau^{2}+3\kappa^{'}\right)-3\tau\tau^{'}+\kappa^{''},\\ \sigma=-\kappa^{2}\tau-\tau^{3}+2\tau\kappa^{'}+\kappa\tau^{'}+\tau^{''}.\end{cases}$$

3.1.3 Spherical Image of the Unit Vector B_{β}^{*}

From the equations (5) and (15) we have

$$\frac{ds_{B}^{*}}{ds^{*}}=\tau^{*}. \tag{16}$$

On the other hand, the following formula is found from the equations (5), (8) and (16).

$$ds_{B}^{*}=\tau^{*}\frac{\sqrt{2\kappa^{2}+\tau^{2}}}{\tau\sqrt{2}}ds_{B} \tag{17}$$

Example 1 Let the curve $\alpha(s)=\left(\frac{4}{5}\sin t,\, 2-\cos t,\, \frac{3}{5}\sin t\right)$ is given. TN-Smarandache curve of this curve is found as

$$\beta(s^*) = \left[\frac{4}{5\sqrt{2}}(\cos t - \sin t), \frac{1}{\sqrt{2}}(\sin t + \cos t), \frac{3}{5\sqrt{2}}(\cos t - \sin t)\right].$$

The spherical images of T^*, N^* and B^* for the curve $\beta(s^*)$ are shown in Figures 1, 2 and 3, respectively.

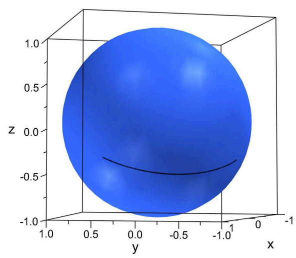

Figure 1 Spherical image of T^*

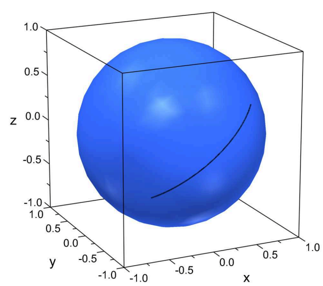

Figure 2 Spherical image of N^*

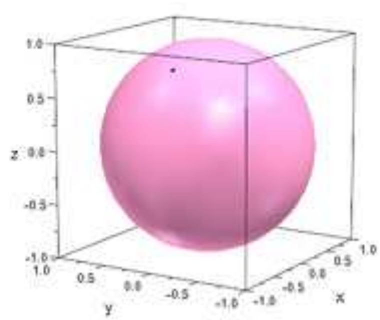

Figure 3 Spherical image of B^*

3.2 NB- Smarandache Curves

Let $\alpha(s)$ be a unit speed regular curve in E^3 and $\{T, N, B\}$ be its moving S. Frenet frame. Smarandache NB curve is defined by ([1])

$$\beta(s^*) = \frac{1}{\sqrt{2}}(N + B). \tag{18}$$

3.2.1 Spherical Image of the Unit Vector T_β^*

From the equations (5) and (8) we have

$$ds^* = \frac{\sqrt{2\kappa^2+\tau^2}}{\kappa\sqrt{2}} ds_T. \tag{19}$$

From the equation (5), we obtain the spherical image of the T_β^* as

$$\frac{ds_T^*}{ds^*} = \kappa^* = \frac{\sqrt{2}\sqrt{\gamma_1^2+\gamma_2^2+\gamma_3^2}}{\left(2\kappa^2+\tau^2\right)^2}, \tag{20}$$

where

$$\begin{cases} \gamma_1 = \kappa\tau\left(2\kappa+\tau'\right)+\tau^2\left(\tau-\kappa'\right), \\ \gamma_2 = -\left[\kappa^2\left(2\kappa^2+3\tau^2+2\tau'\right)+\tau\left(\tau^3-2\kappa\kappa'\right)\right], \\ \gamma_3 = 2\kappa^2\left(\tau'-\tau^2\right)-\tau\left(\tau^3+2\kappa\kappa'\right). \end{cases}$$

Then, the following formula is obtained from the equations (9) and (20).

$$ds_T^* = \frac{\sqrt{\gamma_1^2+\gamma_2^2+\gamma_3^2}}{\kappa\left(2\kappa^2+\tau^2\right)^{3/2}} ds_T \tag{21}$$

3.2.2 Spherical Image of the Unit Vector N_β^*

The spherical image of N_β^* can be found by using the equation (6) as

$$\frac{ds_N^*}{ds^*} = \sqrt{\left(\kappa^*\right)^2+\left(\tau^*\right)^2}, \tag{22}$$

where

$$\tau^* = \frac{\sqrt{2}\left(\kappa\varphi_3+\tau\varphi_1\right)\left(2\tau^2-\kappa^2\right)}{\left(2\tau^3-2\kappa^2\right)^2+\left(\kappa\tau'-\tau\kappa'\right)^2+\left(-\kappa^3+\kappa\tau'-\kappa'\tau\right)^2} \tag{23}$$

and

$$\begin{cases} \varphi_3 = -\tau^3-3\tau\tau'+\kappa^2\tau+\tau'', \\ \varphi_1 = -\kappa^3+\kappa\left(\tau^2+2\tau'\right)+\tau\kappa'-\kappa''. \end{cases}$$

Besides, from the equations (6), (8) and (22)

$$ds_N^* = \sqrt{\left(\kappa^*\right)^2+\left(\tau^*\right)^2}\,\frac{\sqrt{2\kappa^2+\tau^2}}{\sqrt{2}\sqrt{\kappa^2+\tau^2}} ds_N \tag{24}$$

is obtained.

3.2.3 Spherical Image of the Unit Vector B_β^*

From the equations (5) and (23) we have

$$\frac{ds_B^*}{ds^*} = \tau^*. \tag{25}$$

On the other hand, from the equations (5), (8) and (25)

$$ds_B^* = \left[\frac{\sqrt{2}\left(\kappa\varphi_3 + \tau\varphi_1\right)\left(2\tau^2 - \kappa^2\right)}{\left(2\tau^3 - 2\kappa^2\right)^2 + \left(\kappa\tau' - \tau\kappa'\right)^2 + \left(-\kappa^3 + \kappa\tau' - \kappa'\tau\right)^2} \right] \frac{\sqrt{2\kappa^2 + \tau^2}}{\tau\sqrt{2}} ds_B \tag{26}$$

is found.

Example 2 Let the curve $\alpha(s) = \left(\frac{4}{5}\sin t,\ 2 - \cos t,\ \frac{3}{5}\sin t\right)$ is given. NB -Smarandache curve of this curve is

$$\beta(s^*) = \frac{1}{\sqrt{2}}\left[-\frac{4}{5}\sin t - \frac{3}{5},\ \cos t,\ -\frac{3}{5}\sin t + \frac{4}{5}\right].$$

The spherical images of T^* and N^* for the curve $\beta(s^*)$ are shown in Figures 4 and 5, respectively.

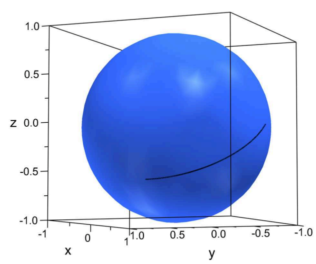

Figure 4 Spherical image of T^*

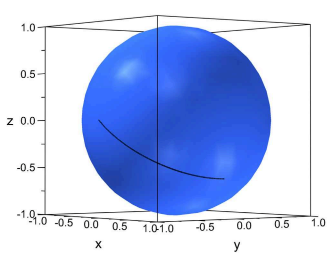

Figure 5 Spherical image of N^*

The spherical image of B^* for the curve $\beta(s^*)$ is a point similar to the Figure 3.

3.3 TB- Smarandache Curves

Let $\alpha(s)$ be a unit speed regular curve in E^3 and $\{T, N, B\}$ be its moving S.Frenet frame. Smarandache TB curve is defined by ([1])

$$\beta(s^*) = \frac{1}{\sqrt{2}}(T + B). \tag{27}$$

3.3.1 Spherical Image of the Unit Vector T_β^*

We can find the spherical image of T_β^* from the equation (5) and obtain

$$\frac{ds_T^*}{ds^*} = \kappa^* = \frac{\sqrt{2}\sqrt{\sigma_1^2+\sigma_2^2+\sigma_3^2}}{\left(2\kappa^2+\tau^2\right)^2}, \tag{28}$$

where

$$\begin{cases} \sigma_1 = \left(2\kappa^2+\tau^2\right)\left(\kappa\tau-\kappa^2\right), \\ \sigma_2 = \left(2\kappa+\tau\right)\left(\kappa^{'}\tau-\kappa\tau^{'}\right), \\ \sigma_3 = \left(2\kappa^2+\tau^2\right)\left(\kappa\tau-\tau^2\right). \end{cases}$$

From the equations (5) and (8)

$$ds^* = \frac{\sqrt{2\kappa^2+\tau^2}}{\kappa\sqrt{2}} ds_T \tag{29}$$

is obtained. Then, the formula following is acquired from equations (28) and (29).

$$ds_T^* = \frac{\sqrt{\sigma_1^2+\sigma_2^2+\sigma_3^2}}{\kappa\left(2\kappa^2+\tau^2\right)^{3/2}} ds_T \tag{30}$$

3.3.2 Spherical Image of the Unit Vector N_β^*

If we use the equation (6) we have the spherical image of N_β^* as

$$\frac{ds_N^*}{ds^*} = \sqrt{\left(\kappa^*\right)^2+\left(\tau^*\right)^2}. \tag{31}$$

Besides, from the equations (6), (8) and (31)

$$ds_N^* = \sqrt{\left(\kappa^*\right)^2+\left(\tau^*\right)^2}\,\frac{\sqrt{2\kappa^2+\tau^2}}{\sqrt{2}\sqrt{\kappa^2+\tau^2}} ds_N \tag{32}$$

is obtained, where

$$\tau^* = \frac{\sqrt{2}\left(\tau-\kappa\right)^2\left(\kappa\Phi_3+\tau\Phi_1\right)}{\left[\tau\left(\kappa-\tau\right)^2\right]^2+\left[\kappa\left(\kappa-\tau\right)^2\right]^2}, \tag{33}$$

$$\begin{cases} \Phi_3 = 2\tau\left(\kappa^{'}-\tau^{'}\right)+\tau^{'}\left(\kappa-\tau\right), \\ \Phi_1 = \kappa^{'}\left(\tau-\kappa\right)+2\kappa\left(\tau^{'}-\kappa^{'}\right). \end{cases}$$

3.3.3 Spherical Image of the Unit Vector B_β^*

From the equations (5) and (33) we have

$$\frac{ds_B^*}{ds^*} = \tau^*. \tag{34}$$

On the other hand, the following formula is found from the equations (5), (8) and (34).

$$ds_B^* = \left[\frac{\sqrt{2}\,(\tau-\kappa)^2\,(\kappa\Phi_3+\tau\Phi_1)}{\left[\tau\,(\kappa-\tau)^2\right]^2+\left[\kappa\,(\kappa-\tau)^2\right]^2} \right] \frac{\sqrt{2\kappa^2+\tau^2}}{\tau\sqrt{2}} ds_B \tag{35}$$

Example 3 Let the curve $\alpha(s) = \left(\frac{4}{5}\sin t,\ 2-\cos t,\ \frac{3}{5}\sin t\right)$ is given. TB -Smarandache curve of this curve is

$$\beta(s^*) = \frac{1}{\sqrt{2}} \left[\frac{4}{5}\cos t - \frac{3}{5},\ \sin t,\ \frac{3}{5}\cos t + \frac{4}{5} \right].$$

The spherical images of T^* and N^* for the curve $\beta(s^*)$ are shown in Figures 6 and 7, respectively.

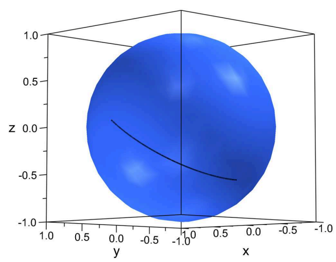

Figure 6 Spherical image of T^*

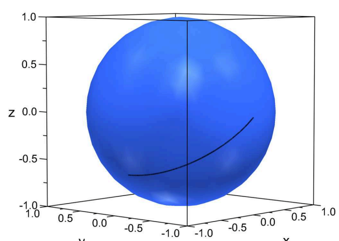

Figure 7 Spherical image of N^*

The spherical image of B^* for the curve $\beta(s^*)$ is a point similar to the Figure 3.

3.4 TNB- Smarandache Curves

Let $\alpha(s)$ be a unit speed regular curve in E^3 and $\{T, N, B\}$ be its moving S.Frenet frame. Smarandache TNB curve is defined by ([1])

$$\beta(s^*) = \frac{1}{\sqrt{3}}(T+N+B). \tag{36}$$

Remark 1 The spherical images of the curve $\beta(s^*)$ can be found in a similar way as presented above.

§4. Spherical Images of Darboux Frame $\{T, g, n\}$

Let S be an oriented surface in E^3. Let $\alpha(s)$ be a unit speed regular curves in E^3 and $\{T, g, n\}$ be Darboux frame of this curve.

4.1 Spherical Image of The Unit Vector T

The differential geometric properties of the spherical image of the unit vector T are given as

$$\frac{dT}{ds_T} = \frac{dT}{ds}.\frac{ds}{ds_T}$$

$$\frac{dT}{ds_T} = (\kappa_g g + \kappa_n n).\frac{ds}{ds_T}$$

$$\frac{ds_T}{ds} = \sqrt{\kappa_g^2 + \kappa_n^2}. \tag{37}$$

On the other hand, from (4) and (37)

$$\kappa = \sqrt{\kappa_g^2 + \kappa_n^2}. \tag{38}$$

can be written.

4.2 Spherical Image of The Unit Vector g

The differential geometric properties of the spherical image of the unit vector g are found as

$$\frac{dg}{ds_g} = \frac{dg}{ds}.\frac{ds}{ds_g}$$

$$\frac{dg}{ds_g} = (-\kappa_g T + \tau_g n).\frac{ds}{ds_g}$$

The relation between the arc length parameters are given as follows

$$\frac{ds_g}{ds} = \sqrt{\kappa_g^2 + \tau_g^2}. \tag{39}$$

4.3 Spherical Image of The Unit Vector n

The differential geometric properties of the spherical image of the unit vector n are given as

$$\frac{dn}{ds_n} = \frac{dn}{ds}.\frac{ds}{ds_n}$$

$$\frac{dn}{ds_n} = (-\kappa_n T - \tau_g g).\frac{ds}{ds_n}$$

Also, the relation between the arc length parameters is obtained as

$$\frac{ds_n}{ds} = \sqrt{\kappa_n^2 + \tau_g^2}. \tag{40}$$

Results:

i) *If $\alpha(s)$ is a geodesic curve, for $\kappa_g = 0$,*

$$\frac{ds_T}{ds} = \kappa_n = \kappa, \qquad \frac{ds_g}{ds} = \tau_g, \qquad \frac{ds_n}{ds} = \sqrt{\kappa_n^2 + \tau_g^2} = \sqrt{\kappa^2 + \tau_g^2}.$$

Also, the unit Darboux vector is as follows

$$c = \frac{\tau_g T + \kappa_n g}{\sqrt{\tau_g^2 + \kappa_n^2}}.$$

ii) *If $\alpha(s)$ is an asymptotic line, for κ_n=0*

$$\frac{ds_T}{ds} = \kappa_g = \kappa, \qquad \frac{ds_g}{ds} = \sqrt{\kappa_g^2 + \tau_g^2} = \sqrt{\kappa^2 + \tau_g^2}, \qquad \frac{ds_n}{ds} = \tau_g,$$

and the unit Darboux vector is

$$c = \frac{\tau_g T + \kappa_g n}{\sqrt{\tau_g^2 + \kappa_g^2}}.$$

iii) *If $\alpha(s)$ is a line of curvature, for τ_g=0*

$$\frac{ds_T}{ds} = \sqrt{\kappa_g^2 + \kappa_n^2} = \kappa, \qquad \frac{ds_g}{ds} = \kappa_g, \qquad \frac{ds_n}{ds} = \kappa_n,$$

and the unit Darboux vector is

$$c = \frac{\kappa_g n + \kappa_n g}{\sqrt{\kappa_g^2 + \kappa_n^2}}.$$

§5. Special Smarandache Curves According To Darboux Frame In E^3

5.1 Tg- Smarandache Curves

Let S be an oriented surface in E^3. Let $\alpha(s)$ be a unit speed regular curve in E^3, $\{T, N, B\}$ and $\{T, g, n\}$ be its S.Frenet frame and Darboux frame, respectively. Smarandache Tg curve is defined by

$$\beta(s^*) = \frac{1}{\sqrt{2}}(T + g). \tag{41}$$

4.2 Tn- Smarandache Curves

Let S be an oriented surface in E^3. Let $\alpha(s)$ be a unit speed regular curve in E^3, $\{T, N, B\}$

and $\{T, g, n\}$ be its S.Frenet frame and Darboux frame, respectively. Smarandache Tn curve is defined by

$$\beta(s^*) = \frac{1}{\sqrt{2}}(T + n). \tag{42}$$

4.3 gn- Smarandache Curves

Let S be an oriented surface in E^3. Let $\alpha(s)$ be a unit speed regular curve in E^3, $\{T, N, B\}$ and $\{T, g, n\}$ be its S.Frenet frame and Darboux frame, respectively. Smarandache gn curve is defined by

$$\beta(s^*) = \frac{1}{\sqrt{2}}(g + n). \tag{43}$$

4.4 Tgn- Smarandache Curves

Let S be an oriented surface in E^3. Let $\alpha(s)$ be a unit speed regular curve in E^3, $\{T, N, B\}$ and $\{T, g, n\}$ be its S.Frenet frame and Darboux frame, respectively. Smarandache Tgn curve is defined by

$$\beta(s^*) = \frac{1}{\sqrt{3}}(T + g + n). \tag{44}$$

(See [2].)

Remark 2 The spherical images of these curves can be easily obtained by the similar way as explained in Section 4.

§6. Conclusion

Spherical mechanisms are very important for robotics. Spherical curves which are drawn by spherical mechanisms are used widely in kinematics and robotics. For this purpose, we presented the spherical images of special Smarandache curves and obtained some relations between them.

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